Imagerie Cérébrale Haute Résolution Basée sur des Solveurs Électromagnétiques Hétérogènes pour des Systèmes de Navigation Intracrânienne en Temps Réel

Hybrid EEG Solvers Enabled High Resolution Brain Imaging for Intracranial Navigation Environments in Real Time

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Mots clés: EEG Haute Résolution, Solveurs Électromagnétiques, Navigation Intracrânienne

L'imagerie cérébrale, cruciale dans de nombreux domaines allant du diagnostic pré-chirurgical pour l'épilepsie aux interfaces cerveau-machine, est très sensible à la précision des modèles électromagnétiques du cerveau et de leurs paramètres physiques. La résolution du processus de reconstruction de l'activité cérébrale peut donc être drastiquement améliorée par une augmentation de la fidélité de ces modèles. Nous proposons une approche basée sur l'agrégation des solutions de plusieurs solveurs innovants à haute-fidélité pour capturer les différents aspects de la réaction des tissus cérébraux aux champs électromagnétiques. Cette approche permet de compenser la variabilité des multiples paramètres des modèles et permet ainsi une reconstruction plus fiable. Les modèles incluent des formulations aux éléments de frontière volumique, surfacique et filaire qui permettent de modéliser, entre autres, l'anisotropie du milieu cérébral. Ces formulations font l'objet d'une étude spectrale en vue de leur préconditionnement avec pour objectif la réduction de la complexité et du coût de leurs processus de résolution lorsqu'elles sont combinées avec des solveurs rapides. Les différents solveurs haute résolution ont été intégrés dans un système de navigation intracrânienne en réalité virtuelle pour permettre d'exploiter, en temps réel, les données obtenues par les différents solveurs avec une lisibilité accrue.

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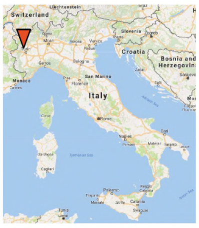
European Research Council
Established by the European Commission

Outline

- Something about us
- Brain and brain imaging
- Computational challenges in brain imaging
- On some recent contributions and applications
- Perspectives for future investigations

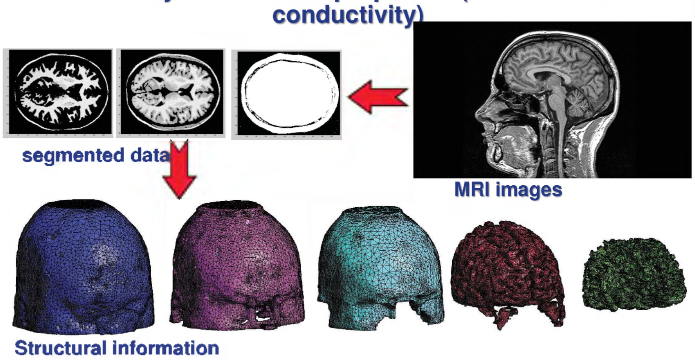
Something about us

Politecnico di Torino (Turin, Italy)



Brain imaging

Structural imaging: provides imaging of brain regions, boundary and material properties (as the electric conductivity)



Brain imaging

Functional imaging: provides imaging of brain *activity*, either electric/magnetic (direct measures, like in EEG or in MEG) or indirect (like in functional MRI, NIRS, or PET)

EEG

MEG

Functional MRI

NIRS

Electroencephalography (EEG)

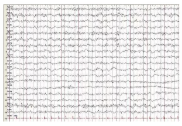
In non-invasive EEG, electrodes are placed on the scalp which measure scalp electric potential generated by neuronal activity

Graphoelements

The EEG output

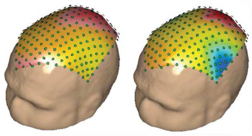
Graphoelements

The voltage in every EEG channel.
Still widely used in the medical practice



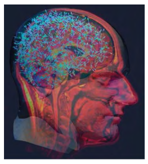
High resolution EEG surface mapping

(Spatial postprocessing, interpolation and filtering of high-density electrodes array EEGs)

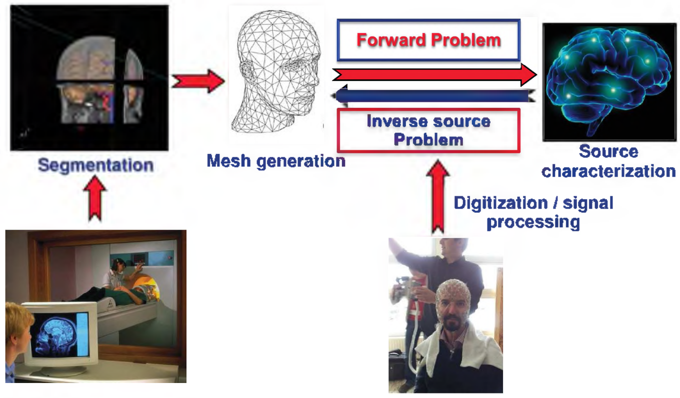


High resolution source reconstructing EEGs

(Spatial postprocessing, volume inverse source from surface high-density EEG data



Summarizing: from scalp potentials to brain currents



Computational Electromagnetics for Brain Research and Applications

Main Challenges from a Computational Prospective

Sources of complexity

- Very low powers involved in the presence of severe bioshielding effects
- Large number of physical degrees of freedom in modeling the microscopic level
- Extremely complex and anisotropic bioelectric physics at the macroscopic level
- Scarce reproducibility of human related parameters and factors

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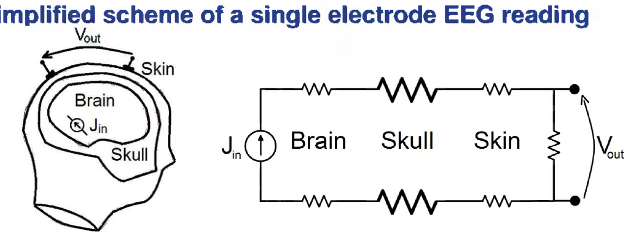
Skull shielding problems

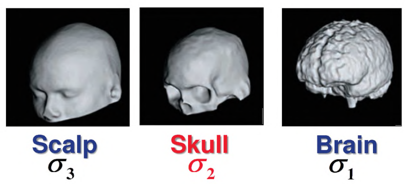
Simplified scheme of a single electrode EEG reading

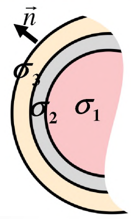


Note that:

- The comparatively lower conductivity of the skull results in a shielding effect for the voltage reading (see invasive EEGs in the following).
- In reality the brain presents a highly inhomogeneous volumetric conductivity which is even anisotropic in certain regions
- Moreover, in normal conditions the brain activity (sources) are not localized (although a localization may occur in certain cases, e.g. during a focal epileptic crisis)

Skull shielding problems





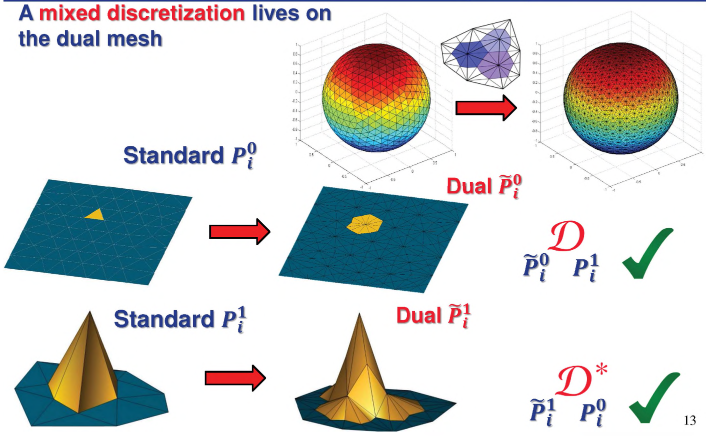
We define the conductivity ratio $\beta = \frac{\sigma_1}{\sigma_2}$ $\quad \sigma_1 \approx \sigma_3$

$\beta \to 1$ ➡ Optimal performance

$\beta \to \infty$ ➡ **Degradation of performance**

Mixed discretizations in EEG BEMs

A mixed discretization lives on the dual mesh

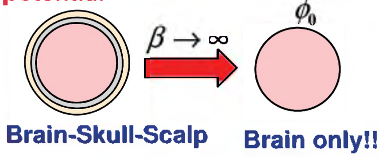


Skull shielding problem

Two source of errors

- Scaling difference in the potential

$$\phi \propto \left\{\phi_0, \frac{\sigma_1}{\beta}, \frac{\sigma_1}{\beta}\right\}$$



- Numerical cancellation

$$\phi(\boldsymbol{r}) = v_{\text{dip}}(\boldsymbol{r}) + \phi_{\text{h}}(\boldsymbol{r}) \qquad \phi_{\text{h}} = \sum_{i=1}^{N} \mathcal{S}_{ji} q_{\Gamma_i} \quad \text{for } i = 1, \ldots, N\,.$$

$$\lim_{\beta\to\infty} \phi_{\text{h}}|_{\text{scalp}} \to -v_{\text{dip}}|_{\text{scalp}}$$ → Catastrophic cancellation

Indirect Adjoint Double Layer formulation

IADL: New Integral formulation → high accuracy independently of β

- The electric potential is harmonic in the outermost layer

$$\Delta\phi = 0$$

- Write the potential as a contribution of monopole sources $J\,|_{\Gamma 2}$ and $J\,|_{\Gamma 3}$.

$$\phi(\boldsymbol{r}) = \mathcal{S}J|_{\Gamma_2}(\boldsymbol{r}) + \mathcal{S}J|_{\Gamma_3}(\boldsymbol{r})$$

- Applying the gradient operator in the normal direction

$$\partial_{\hat{n}}\phi|_{\Gamma_j} = \pm\frac{J_{\Gamma_j}}{2} + \mathcal{D}^*_{j2}J_{\Gamma_2} + \mathcal{D}^*_{j3}J_{\Gamma_3}$$

Indirect Adjoint Double Layer formulation

- The operator $\mathcal{S}u|_{\Gamma_i}$ is continuous across and interface

$$\to [\phi] = 0 \quad ✓$$

- The operator $\mathcal{D}^*u|_{\Gamma_i}$ is discontinuous across and interface

$$\boldsymbol{n}\cdot\sigma_j\nabla\phi|^-_{\Gamma_j} = \boldsymbol{n}\cdot\sigma_{j+1}\nabla\phi|^+_{\Gamma_j} \quad ✗$$

This leads to the following

$$\boldsymbol{n}\cdot\nabla\phi|_{\Gamma 3} = 0 \;\to\; \partial_{\hat{n}}\phi|_{\Gamma_3} = 0$$

$$[n\cdot\sigma\nabla\phi] = 0 \;\to\; \partial_{\hat{n}}\phi|_{\Gamma_2} = -\frac{1}{2}q_{\Gamma 2} + \sum_{i=1}^{N}\mathcal{D}^*_{2i}q_{\Gamma_i} + \partial_{\hat{n}}v_{\text{s}1}|_{\Gamma_2} = \frac{\sigma_2}{\sigma_3-\sigma_2}q_{\Gamma 2}$$

Indirect Adjoint Double Layer formulation

Finally, we get the following equation

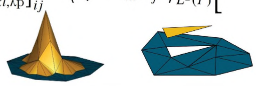
$$-\frac{J_{\Gamma_2}}{2} + D^*_{22}J_{\Gamma_2} + D^*_{23}J_{\Gamma_3} = \frac{\sigma_2}{\sigma_3-\sigma_2}q|_{\Gamma_2}$$

$$\frac{J_{\Gamma_3}}{2} + D^*_{32}J_{\Gamma_2} + D^*_{33}J_{\Gamma_3} = 0$$

The discretized system

$$\left[I_{k,\tilde{\lambda}p}\right]_{ij} = \left\langle\tilde{\lambda}_i^{(k)}, p_j^{(k)}\right\rangle_{L^2(\Gamma)} \qquad \left[D^*_{kl,\tilde{\lambda}p}\right]_{ij} = \left\langle\tilde{\lambda}_i^{(k)}, \mathcal{D}^*_{kl}\,p_j^{(l)}\right\rangle_{L^2(\Gamma)}$$

$$\begin{bmatrix} -\frac{1}{2}I_{22} + D^*_{22} & D^*_{23} \\ D^*_{32} & \frac{1}{2}I_{33} + D^*_{33} \end{bmatrix}\begin{bmatrix} J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} \frac{\sigma_2}{\sigma_3-\sigma_2}q_2 \\ 0 \end{bmatrix}$$



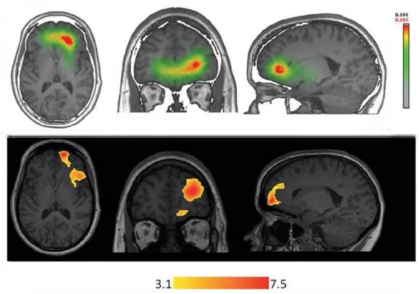
- No additional computation cost in building the equation.
- Higher accuracy even for high conductivity ratio.

Target Applications (I)

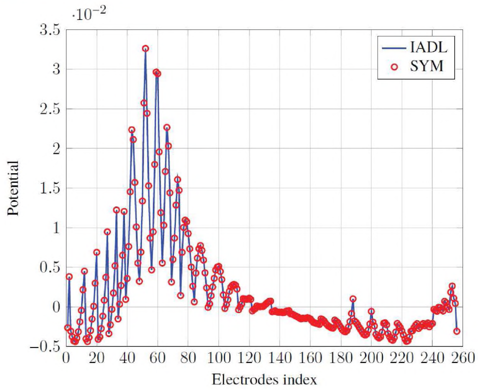
Epileptogenic Area Localization in Focal Epilepsy

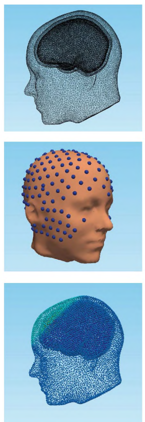
Task: localizing characterizing the brain electric sources during an epileptic crisis

Societal impact: it's a key pre-surgical step and could minimize the use of invasive EEG techniques requiring skull's trepanation



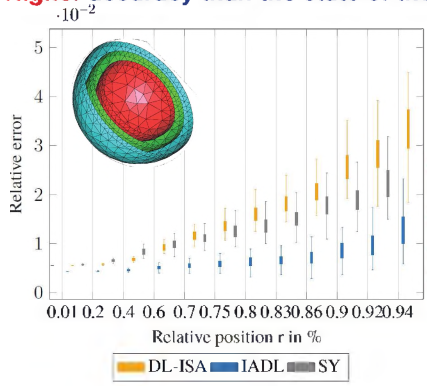
Impact

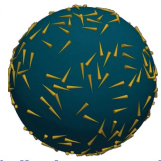




Impact

Higher accuracy than the state of the art





- 500 dipole sources for each eccentricity.
- Random position.
- Radial orientation.
- Random tangential orientation.

Computational Electromagnetics for Brain Research and Applications

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Computational Electromagnetics for Brain Research and Applications

Source of complexity

Large number of physical degrees of freedom in modeling the microscopic level

Our strategy

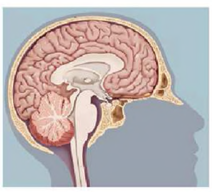
New computational paradigms that operates in linear-instead-of-cubic complexity

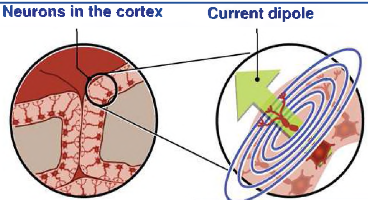
Two approaches to linear complexity operations

direct — Fast Matrix Inversions (fast inverse solvers)

iterative — Fast matrix-vector products + analytical regularizations

The Forward EEG problem





$$J(r) = P\delta(r - r')$$

Frequency <100Hz

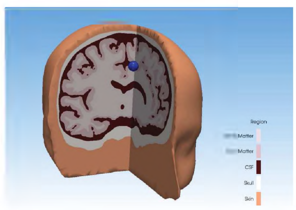
Quasi-static approximation of Maxwell equations

$$\nabla . \bar{\bar{\sigma}}(r)\Phi(r) = \nabla \bullet J(r)$$

Poisson's equation

Computational Electromagnetics for Brain Research and Applications

If you want to find neuronal currents in the brain, you may think to invert the matrix below.



$$\mathbf{Z} = \begin{bmatrix} (\sigma_1+\sigma_2)\mathbf{N}_{11} & -2\mathbf{D}^*_{11} & -\sigma_2\mathbf{N}_{12} & \mathbf{D}^*_{12} & 0 & 0 & 0 \\ -2\mathbf{D}_{11} & (\sigma_1^{-1}+\sigma_2^{-1})\mathbf{S}_{11} & \mathbf{D}_{12} & -\sigma_2^{-1}\mathbf{S}_{12} & 0 & 0 & 0 \\ -\sigma_2\mathbf{N}_{21} & \mathbf{D}^*_{21} & (\sigma_2+\sigma_3)\mathbf{N}_{22} & -2\mathbf{D}^*_{22} & -\sigma_3\mathbf{N}_{23} & \mathbf{D}^*_{23} & 0 \\ \mathbf{D}_{21} & -\sigma_2^{-1}\mathbf{S}_{21} & -2\mathbf{D}_{22} & (\sigma_2^{-1}+\sigma_3^{-1})\mathbf{S}_{22} & \mathbf{D}_{23} & -\sigma_3^{-1}\mathbf{S}_{23} & 0 \\ 0 & 0 & -\sigma_3\mathbf{N}_{32} & \mathbf{D}^*_{32} & (\sigma_3+\sigma_4)\mathbf{N}_{33} & -2\mathbf{D}^*_{33} & \cdots \\ 0 & 0 & \mathbf{D}_{32} & -\sigma_3^{-1}\mathbf{S}_{32} & -2\mathbf{D}_{33} & (\sigma_3^{-1}+\sigma_4^{-1})\mathbf{S}_{33} & \cdots \\ 0 & 0 & 0 & 0 & \vdots & \vdots & \ddots \end{bmatrix}$$

don't! That's cubic!

Computational Electromagnetics for Brain Research and Applications

$$\mathbf{Z}=\begin{bmatrix}(\sigma_1+\sigma_2)\,\mathbf{N}_{11} & -2\mathbf{D}^*{}_{11} & -\sigma_2\mathbf{N}_{12} & \mathbf{D}^*{}_{12} & 0 & 0 & 0\\ -2\mathbf{D}_{11} & (\sigma_1^{-1}+\sigma_2^{-1})\,\mathbf{S}_{11} & \mathbf{D}_{12} & -\sigma_2^{-1}\mathbf{S}_{12} & 0 & 0 & 0\\ -\sigma_2\mathbf{N}_{21} & \mathbf{D}^*{}_{21} & (\sigma_2+\sigma_3)\,\mathbf{N}_{22} & -2\mathbf{D}^*{}_{22} & -\sigma_3\mathbf{N}_{23} & \mathbf{D}^*{}_{23} & 0\\ \mathbf{D}_{21} & -\sigma_2^{-1}\mathbf{S}_{21} & -2\mathbf{D}_{22} & (\sigma_2^{-1}+\sigma_3^{-1})\,\mathbf{S}_{22} & \mathbf{D}_{23} & -\sigma_3^{-1}\mathbf{S}_{23} & 0\\ 0 & 0 & -\sigma_3\mathbf{N}_{32} & \mathbf{D}^*{}_{32} & (\sigma_3+\sigma_4)\,\mathbf{N}_{33} & -2\mathbf{D}^*{}_{33} & \cdots\\ 0 & 0 & \mathbf{D}_{32} & -\sigma_3^{-1}\mathbf{S}_{32} & -2\mathbf{D}_{33} & (\sigma_3^{-1}+\sigma_4^{-1})\,\mathbf{S}_{33} & \cdots\\ 0 & 0 & 0 & 0 & \vdots & \vdots & \ddots\end{bmatrix}$$

To get it done linearly, pre-multiply it instead with its dual magneto-to-electric and electro-to-magnetic counterpart

$$\bar{\mathbf{C}}=\begin{bmatrix}c_{11}\bar{\mathbf{S}}_{11} & c_{12}\bar{\mathbf{D}}_{11} & c_{13}\bar{\mathbf{S}}_{12} & c_{14}\bar{\mathbf{D}}_{12} & 0 & 0 & 0\\ c_{21}\bar{\mathbf{D}}^*_{11} & c_{22}\bar{\mathbf{N}}_{11} & c_{23}\bar{\mathbf{D}}^*_{12} & c_{24}\bar{\mathbf{N}}_{12} & 0 & 0 & 0\\ c_{31}\bar{\mathbf{S}}_{21} & c_{32}\bar{\mathbf{D}}_{21} & c_{33}\bar{\mathbf{S}}_{22} & c_{34}\bar{\mathbf{D}}_{22} & c_{35}\bar{\mathbf{S}}_{23} & c_{36}\bar{\mathbf{D}}_{23} & 0\\ c_{41}\bar{\mathbf{D}}^*_{21} & c_{42}\bar{\mathbf{N}}_{21} & c_{43}\bar{\mathbf{D}}^*_{22} & c_{44}\bar{\mathbf{N}}_{22} & c_{45}\bar{\mathbf{D}}^*_{23} & c_{46}\bar{\mathbf{N}}_{23} & 0\\ 0 & 0 & c_{53}\bar{\mathbf{S}}_{32} & c_{54}\bar{\mathbf{D}}_{32} & c_{55}\bar{\mathbf{S}}_{33} & c_{56}\bar{\mathbf{D}}_{33} & \cdots\\ 0 & 0 & c_{63}\bar{\mathbf{D}}^*_{32} & c_{64}\bar{\mathbf{N}}_{32} & c_{65}\bar{\mathbf{D}}^*_{33} & c_{66}\bar{\mathbf{N}}_{33} & \cdots\\ 0 & 0 & 0 & 0 & \vdots & \vdots & \ddots\end{bmatrix}$$

Computational Electromagnetics for Brain Research and Applications

To get it done linearly, pre-multiply it instead with its dual magneto-to-electric and electro-to-magnetic counterpart

In fact the following identities can be proved

$$S_{ii}N_{ii}=\frac{1}{4}I-D_{ii}^2$$
$$N_{ii}S_{ii}=\frac{1}{4}I-D_{ii}^{*2}$$

and the product reads

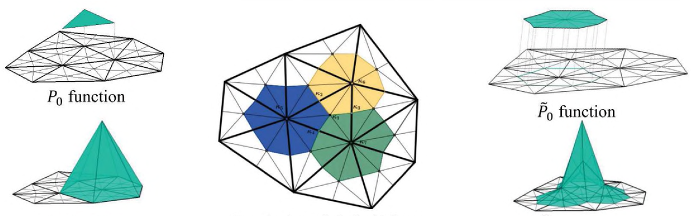
$$CZ=\begin{bmatrix}(\sigma_1+\sigma_2)^2\boxed{S_{11}N_{11}}+K_{11} & K_{12} & K_{13} & \cdots\\ K_{21} & (\sigma_1^{-1}+\sigma_2^{-1})^2\boxed{N_{11}S_{11}}+K_{22} & K_{23} & \cdots\\ K_{31} & K_{32} & (\sigma_2+\sigma_3)^2\boxed{S_{22}N_{22}}+K_{33} & \cdots\\ \vdots & \vdots & \vdots & \ddots\end{bmatrix}$$

This basically gives you points 1) and 2) of the general recipe

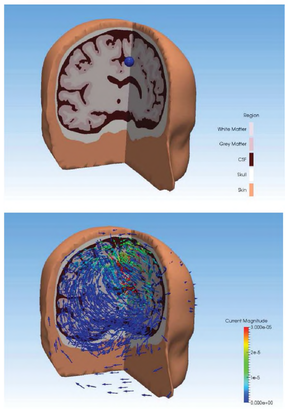
Point 3) is obtained by discretizing things properly…

Computational Electromagnetics for Brain Research and Applications

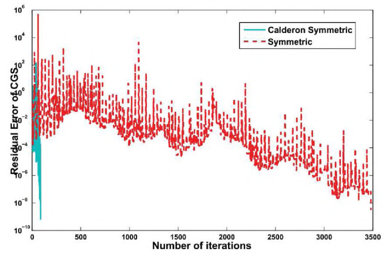
Point 3) is obtained by permeating the discretization choice by the "electromagnetic duality": electric unknowns on one graph, magnetic ones on a dual graph…



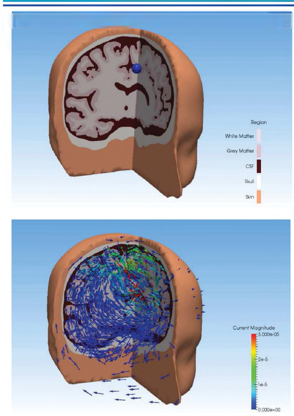
Computational Electromagnetics for Brain Research and Applications



The result of the overall process ensures orders of magnitude accelerations and a change from a cubic to linear computational complexity!



Computational Electromagnetics for Brain Research and Applications



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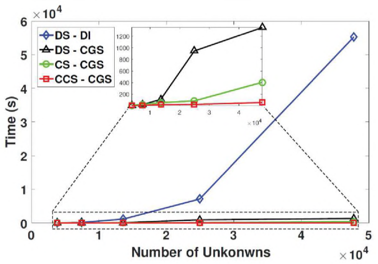
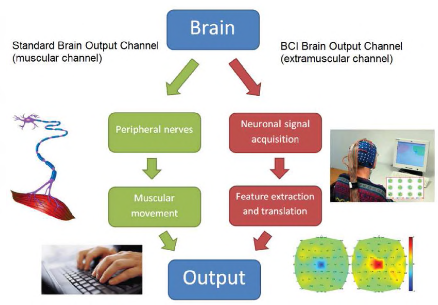


Target Applications (II)

Brain Computer Interfaces

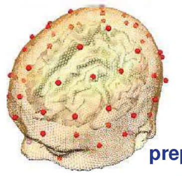
Task: create an extra-muscular channel to connect the Brain with the external world

Societal impact: BCIs are a key resource for locked-in patients and are also of increasing interest for general public applications



Brain Computer Interfaces

Standard pipeline



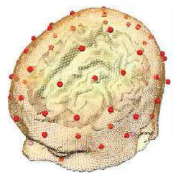
EEG preprocessing → Output data → feature extraction → Feature vector → classification → Classified output

Reduction of the order of degrees of freedom (to get a lower dimensional feature vector): power spectrum, autoregression, and independent component analyses

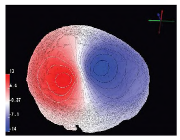
Classifiers assign a class to a given feature vector: Linear discriminant analysis, Support Vector Machine, Neural networks, etc...

Brain Computer Interfaces

Standard pipeline – generating output data



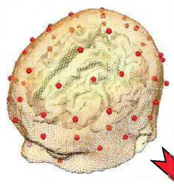
EEG → Surface voltage → filtering → Output data

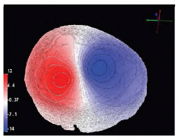


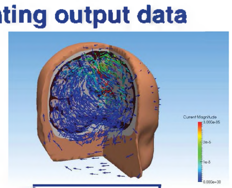
We can extend this chain...

Brain Computer Interfaces

Our work: extended pipeline – generating output data



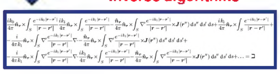


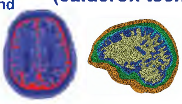


EEG → Surface voltage → Volumetric current → volume filtering → Output data

"impedance" inversion (calderon technology)

Volume conduction propagators and inverse algorithms





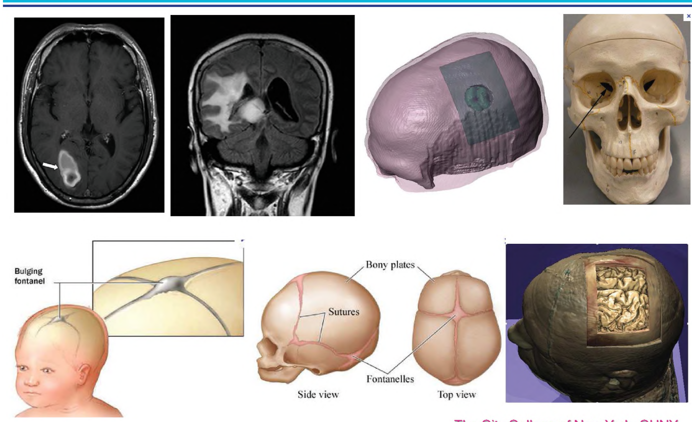
Computational Electromagnetics for Brain Research and Applications

Main Challenges from a Computational Prospective

Sources of complexity

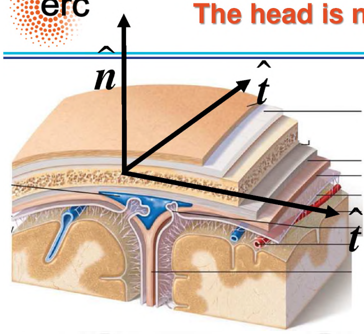
- Very low powers involved in the presence of severe bioshielding effects
- Large number of physical degrees of freedom in modeling the microscopic level
- Extremely complex and anisotropic bioelectric physics at the macroscopic level
- Scarce reproducibility of human related parameters and factors

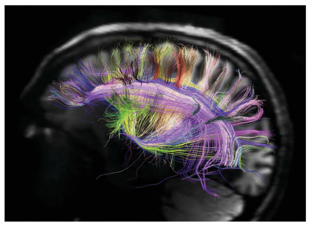
The head is not homogeneous



The City College of New York, CUNY

The head is not isotropic

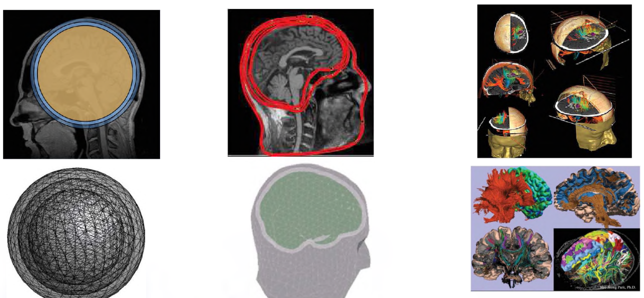




$$\overline{\overline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

The electric conductivity is, in general, a varying-in-space tensor

Head Models



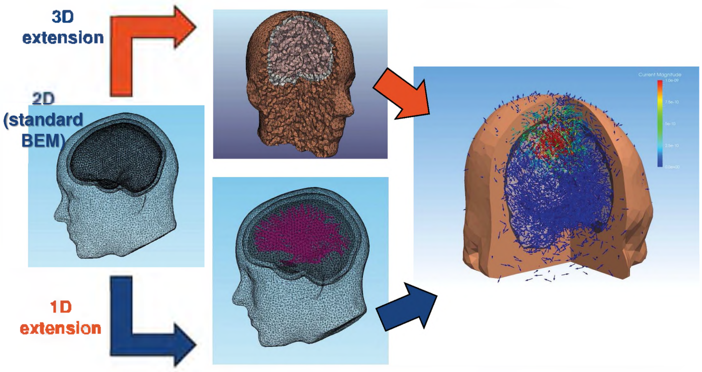
Spherical head models
- Analytical solution
- Fast computation
- limited to unrealistic geometries

Surfacic head models
- Integral based method (BEM)
- Incorporating realistic geometry

Volumetric head models
- Differential based method (FEM, FDM)
- Incorporating detailed geometry
- Incorporating more complex heterogeneity of the different tissues and anisotropic conductivity

Computational Electromagnetics for Brain Research and Applications

We have obtained integral equation 3D-2D-1D hybrid that can naturally be adapted to MRI data

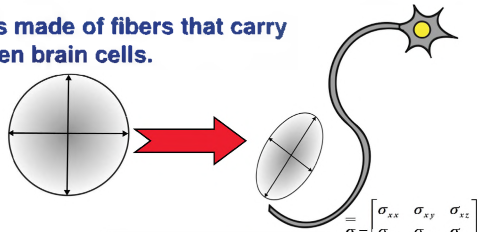


The brain conductivity is anisotropic and inhomogeneous

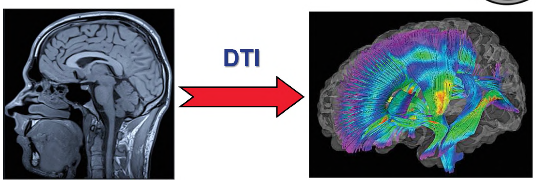
- **Anisotropies:**

The white matter is made of fibers that carry information between brain cells.

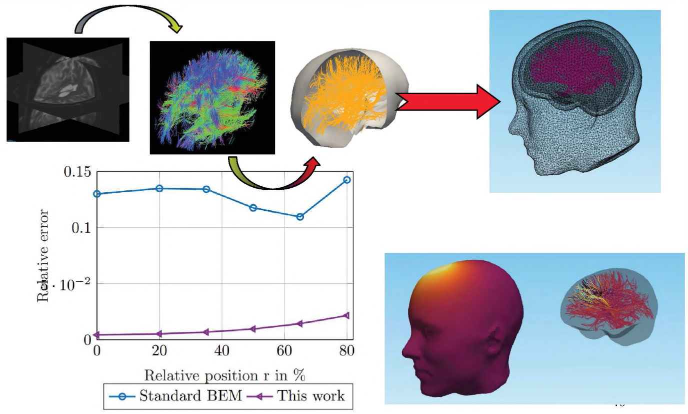
$$\bar{\bar{\sigma}} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}$$



$$\bar{\bar{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$



Impact

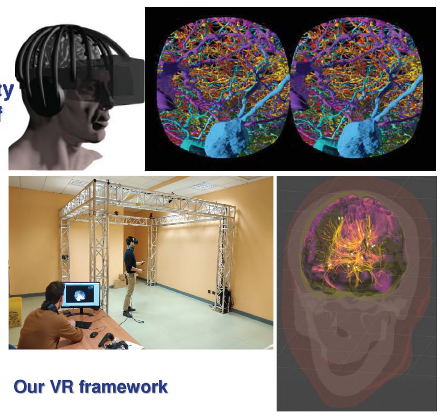


Target Applications (III)

Neurofeedback

Task: create real time displays of neuroactivity to teach patients to self regulate his/her own brain functions

Societal impact: it enables neurotherapy that is increasingly considered as part of therapeutic strategies of anxiety, depression, attention deficit and obsessive compulsive disorders



Our VR framework

Conclusions and Perspectives for future investigations

- This talk delineated some investigation axes in computational science for brain research
- Computationally intensive paradigms enables promising paths in brain imaging and applications
- Our current and future investigations include the translation of our strategies for MEG and to active techniques.
- In these efforts we acknowledge our ERC project in computational electromagnetics (ERC CoG 321, Grant N° 724846) which has been supporting us since September 2017.