Gravitational wave searches using radio pulsar timing
Recherche des ondes gravitationnelles par la chronométrie des pulsars

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Abstract:

Pulsars are rapidly-rotating, highly-magnetized neutron stars both in supernova explosions of massive stars. They produce beams of radio emission that are swept across the sky as they rotate, so that pulsars appear to us as "cosmic lighthouses". The fastest-spinning pulsars, the so-called "millisecond pulsars" (MSPs), have spin periods as short as a few milliseconds and are extremely stable rotators so that the weak radio pulses of an MSP are analogous to the ticks of an ultra-stable clock located at a very large distance. By combining such ticks from an ensemble of MSPs distributed across the sky, it is possible to search for very low-frequency (nHz) gravitational waves (GWs), e.g. emitted by supermassive black hole binaries at cosmological distances. I will present a review of pulsars observations in radio, in the context of GW searches with pulsar timing.

Résumé:

Les pulsars sont des étoiles à neutrons fortement magnétisées et en rotation rapide, nés lors de l'explosion d'étoiles massives en supernovae. Ils produisent des faisceaux d'émission qui balaient le ciel au fil de la rotation de l'étoile, si bien qu'ils nous apparaissent comme des "phares cosmiques". Ceux dont la rotation est la plus rapide, les pulsars milliseconde (MSPs), ont des périodes de rotation de quelques millisecondes seulement. Leur rotation est si stable que les faibles impulsions radio émises par un MSP sont analogues aux battements d'une horloge ultra-stable située à une très grande distance. En combinant les battements d'un ensemble de MSPs bien répartis sur le ciel, il est possible de chercher des ondes gravitationnelles (OGs) de très basses fréquences (nHz), émises par exemple par des systèmes binaires de trous noirs supermassifs, à des distances cosmologiques. Je vais présenter une revue des observations de pulsars en radio, dans le contexte des recherches d'OGs par la chronométrie des pulsars.

1 Introduction: Pulsar Timing

A few years ago, the first detection of gravitational waves (GWs) emitted by coalescing binary black holes by the LIGO-VIRGO interferometers opened the era of GW astrophysics [1]. These ground based detectors are sensitive to GWs with frequencies ranging from a few Hz to several kHz or more. LISA, the GW space observatory, will be launched around 2034 and will be sensitive to GWs in the $10^{-6} - 10^{-1}$ Hz range [2]. At even lower frequencies ($10^{-9} - 10^{-5}$ Hz), Pulsar Timing Arrays (PTAs) use sets of ultra-stable pulsars distributed across the sky to search for GWs emitted, for instance, by supermassive black hole binaries at cosmological distances. In this review we first present the principles of the "pulsar timing" technique, and then give an overview of low-frequency GW searches using PTAs. We finally discuss the current limitations of PTAs and avenues for overcoming these limitations.

Pulsars are rapidly rotating, highly magnetized neutron stars born in supernova explosions of massive stars at the end of their lives. They emit beams of electromagnetic radiation which are swept across the sky as they rotate, in the same way lighthouse beams sweep across an observer, so that they appear to pulse to a distant observer. Since the first discovery of a pulsar in 1967 [3], over 2600 pulsars have now been detected, mainly from radio observations[1]. Most known pulsars are in the Galactic disk, and a fraction reside in globular clusters or in the Magellanic clouds. The majority of known pulsars are split into two categories: "Normal pulsars", which have rotational periods P ranging from 0.1 s to ∼10 s, and the so-called "millisecond pulsars" (MSPs) which have periods between 1.4 ms to a few tens of ms and are thought to have been spun-up by the accretion of matter and angular momentum from a companion in a binary system [4]. In practice, most known MSPs are indeed observed to be in binary systems. Radio observations of MSPs have demonstrated that they are extremely stable rotators, that can be used for a large variety of astrophysical applications. Timing observations of the binary pulsar PSR B1913+16 provided the first observational evidence for GWs [5]. Observations of the millisecond pulsar B1257+12 enabled the very first discovery of exoplanets [6]. The long-term timing of the pulsars in the double pulsar system PSR J0737−3039A/B provides stringent tests of theories of gravity in

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[1] See ATNF Pulsar Catalogue, `http://www.atnf.csiro.au/research/pulsar/psrcat/expert.html`

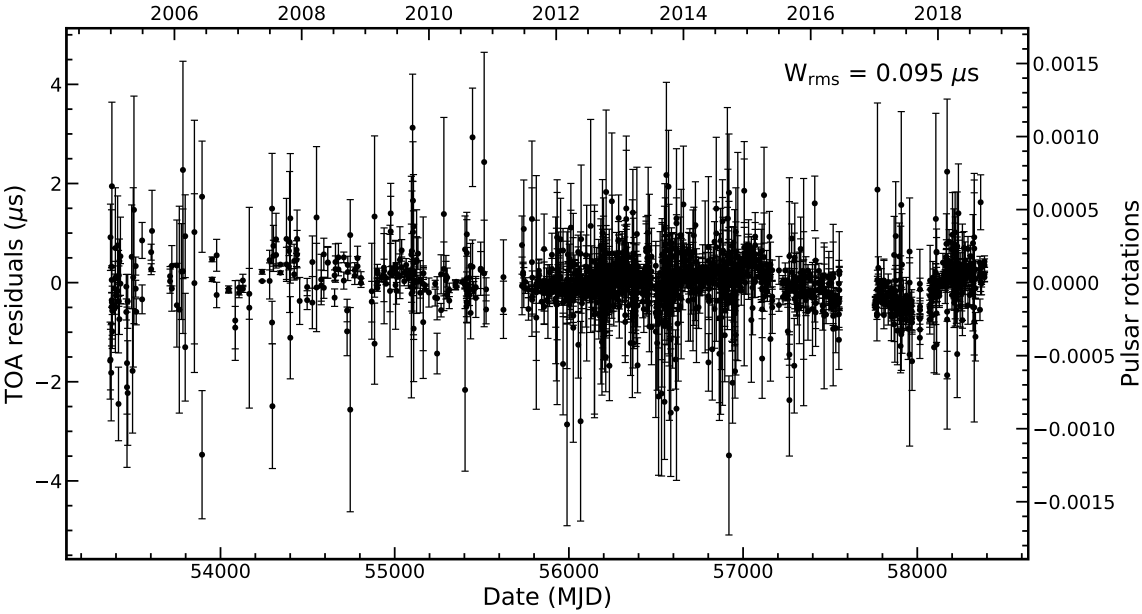

Figure 1 – Nançay Radio Telescope timing residuals for the millisecond pulsar J1909−3744, between 2005 and early 2018. The bulk of the observations were conducted at a frequency of 1.4 GHz, with a bandwidth of 128 MHz for observations made before mid-2011, and a bandwidth of 512 MHz for those made after that date. The RMS of the timing residuals is ~ 95 *ns, i.e., our timing model for PSR J1909−3744 enables us to predict TOAs with a typical accuracy below 100 ns.*

the strong-field regime [7], and the recently-discovery MSP in a triple system, PSR J0337+1715, is a unique laboratory for testing the strong equivalence principle of General Relativity [8].

Pulsar Timing consists in measuring the times of arrival (TOAs) of the pulses of a pulsar at a telescope, and comparing these TOAs with those predicted by a "timing model". Since individual pulses are generally too weak to be detected, pulse profiles are typically formed by averaging up minutes to hours of observation, and the profiles are then compared to a "standard profile" for the pulsar to extract the TOAs. These topocentric TOAs are then transferred to an inertial reference frame, the solar system barycenter, to eliminate effects caused by the motion of the Earth with respect to the pulsar. The timing data are also "dedispersed", *i.e.*, they are corrected for the frequency-dependent dispersion delays caused by the presence of ionized gas between the pulsar and the telescope. For pulsars residing in binary systems, the barycentric TOAs are then corrected for the orbital motion. Finally, the TOAs transferred to the pulsar frame and the pulsar's rotational parameters (rotational period P and subsequent time derivatives, $\dot{P}$, $\ddot{P}$, etc.) are used to calculate rotational phases $\Phi(t)$ as a function of time, which are compared with the predictions of the timing model to form "timing residuals", $R(t)$, calculated as:

$$R(t_i) = P(t_i) \times [\Phi(t_i) - N(t_i)] \,, \tag{1}$$

where t_i denotes the i-th TOA, $P(t_i)$ is the rotational period at t_i, $\Phi(t_i)$ the rotational phase at t_i, and $N(t_i)$ is the nearest integer to $\Phi(t_i)$. Figure 1 shows Nançay Radio Telescope timing residuals over several years for the MSP J1909−3744 (rotational period $P \sim 2.947$ ms, orbital period $P_{\rm orb} \sim 1.533$ d), the latter MSP being the most stable pulsar visible from Nançay. As can be seen from the above expression, what makes pulsar timing powerful is that it makes it possible to unambiguously account for every single rotation of a given pulsar over long periods of time. Timing models include the pulsar's rotational parameters mentioned above, its astrometric parameters (sky coordinates, proper motion parameters, etc.), dispersion parameters (dispersion measure DM and time derivatives), and the orbital parameters for pulsars in binary systems. The timing residuals are minimized in a least-square fitting procedure to estimate the parameters of the timing model and their uncertainties, using dedicated pulsar timing software such as TEMPO2[2] [9]. Unmodelled, systematic variations in the timing residuals can indicate the existence of errors in the model, or the presence of effects affecting the timing, that need to be accounted for. The latter effects can for instance include unmodelled noise in the pulsar timing, post-Newtonian perturbations to the pulsar's orbital motion, or the signature of gravitational waves.

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[2] `https://sourceforge.net/projects/tempo2/`

2 Gravitational wave searches with PTAs

As mentioned above, MSPs are extremely stable rotators, and they are thus similar to clocks located at large (typically, galactic) distances that can be used for a number of astrophysical applications. Nevertheless, since the timing of a given MSP can still be affected by unmodelled intrinsic effects, timing noise due to changes of the interstellar medium or other sources of noise (discussed later in this article), increased sensitivity to timing perturbations caused by the passage of GWs can be achieved by observing an array of stable MSPs, widely distributed across the sky and over as long a time period as possible, and by searching for correlations in the timing residuals of these pulsars. In these PTA experiments, the pulsars are test masses, and each pulsar-Earth baseline acts as the arm of giant, Galactic-scale interferometer. In practice, the passage of a GW signal will cause the observed pulse frequency ν of a given pulsar to fluctuate, by:

$$\frac{\partial\nu}{\nu} = -H^{ij}\left[h_{ij}(t_e, x_e^i) - h_{ij}(t_e - D/c, x_p^i)\right], \tag{2}$$

where D is the distance to the pulsar, c is the speed of light, h_{ij} denotes the GW strain evaluated at the Earth at time t_e and position x_e, and at the pulsar, and H^{ij} is a geometrical term that depends on the positions of the GW source, of the Earth and of the pulsar. This rotational frequency will in turn induce fluctuations in the timing, *i.e.*, timing residuals. For example, the amplitude of the timing residuals caused by a distant binary system can be estimated to be [10]:

$$\Delta R(t) = 10\ \mathrm{ns}\left(\frac{1\mathrm{Gpc}}{d}\right)\left(\frac{M}{10^9 M_\odot}\right)^{5/3}\left(\frac{10^{-7}\ \mathrm{Hz}}{f}\right)^{1/3}. \tag{3}$$

In the above expression, d is the distance to the system, f is the GW frequency and $M/(1+z)$ (where z is the redshift) is the total mass of the system.

PTA observations are typically conducted with a sampling interval of days or weeks over $T = 10$ yrs or more, implying that they are sensitive to GWs with frequencies in the order of $1/T \sim 1 - 100$ nHz. In this GW frequency range, potentially detectable sources could be: individual supermassive black hole binary systems, a stochastic background emitted by a population of supermassive black hole binaries distributed throughout the Universe, GW signals from cosmic strings [11] and inflation [12], bursts with memory caused by black hole mergers [13], or other sources of GW bursts. In the case of an isotropic, stochastic background of GWs from a population of supermassive black hole binaries, the timing residuals for pairs of pulsars will be correlated as [14]:

$$\chi(\zeta) = \frac{3}{2}x\ln x - \frac{x}{4} + \frac{1+\delta(x)}{2} \tag{4}$$

with $x = \dfrac{1-\cos(\zeta)}{2}$. Here, ζ is the sky separation between the two pulsars and δ is the Dirac delta function. This correlation function, which is displayed in Figure 2, is known as the Hellings-and-Downs curve.

Three major PTAs are currently actively searching for low-frequency gravitational waves. The Parkes PTA (PPTA, [16]) uses pulsar timing data from the Parkes radio telescope in Australia. The North American Nanohertz Observatory for Gravitational Waves (NANOGrav, [17]) uses pulsar observations made with the Green Bank telescope in the US and with Arecibo telescope in Puerto Rico. Finally, the European PTA (EPTA, [18]) uses pulsar timing data recorded with the Jodrell Bank (UK), Westerbork (Netherlands), Effelsberg (Germany), Sardinia (Italy) and Nançay (France) radio telescopes. Each PTA typically has timing data on a few tens of MSPs over time periods of a few years to several decades. The three PTAs have recently joined forces and combined data to form an International PTA (IPTA, [19]), whose sensitivity is higher than those of individual PTAs. Data from the five European telescopes are also combined coherently, to simulate pulsar observations with a much higher effective aperture, equivalent to a 195-m diameter telescope (the Large European Array for Pulsars, LEAP, see [18]). In the future, it is expected that radio telescopes located in other countries or regions will join the IPTA and contribute to GW searches. Examples include the Chinese Five Hundred Metre Aperture Spherical Telescope (FAST) that recently started operating, the future 110-m diameter fully-steerable single-dish radio telescope QTT, also in China, MeerKAT in South Africa and the planned Square Kilometre Array (SKA). Telescopes that are currently in the IPTA and those that are expected to join in the future are shown in Figure 3.

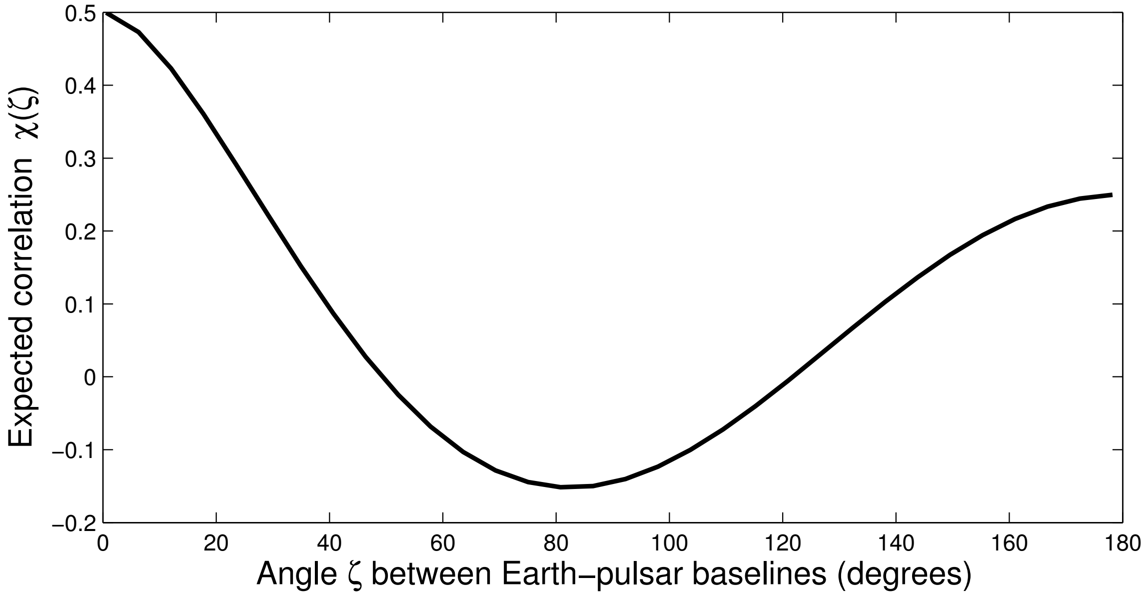

Figure 2 – The Hellings-and-Downs curve, which corresponds to the expected correlation between the timing residuals of pairs of pulsars due to an isotropic stochastic background of GWs, as a function of the angular separation between the pulsars in the pairs. Figure taken from [15].

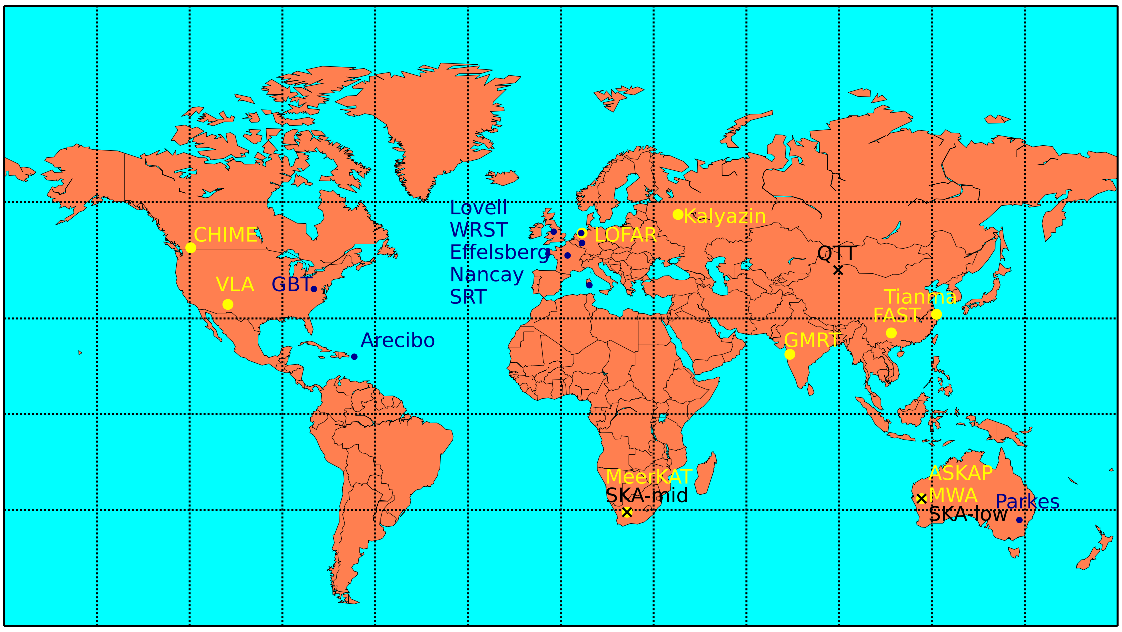

Figure 3 – Map showing the locations of the radio telescopes that are currently part of the International Pulsar Timing Array (IPTA), and those that are expected to join in the future. Telescopes that are already in the IPTA are labelled in blue. Figure adapted from [20].

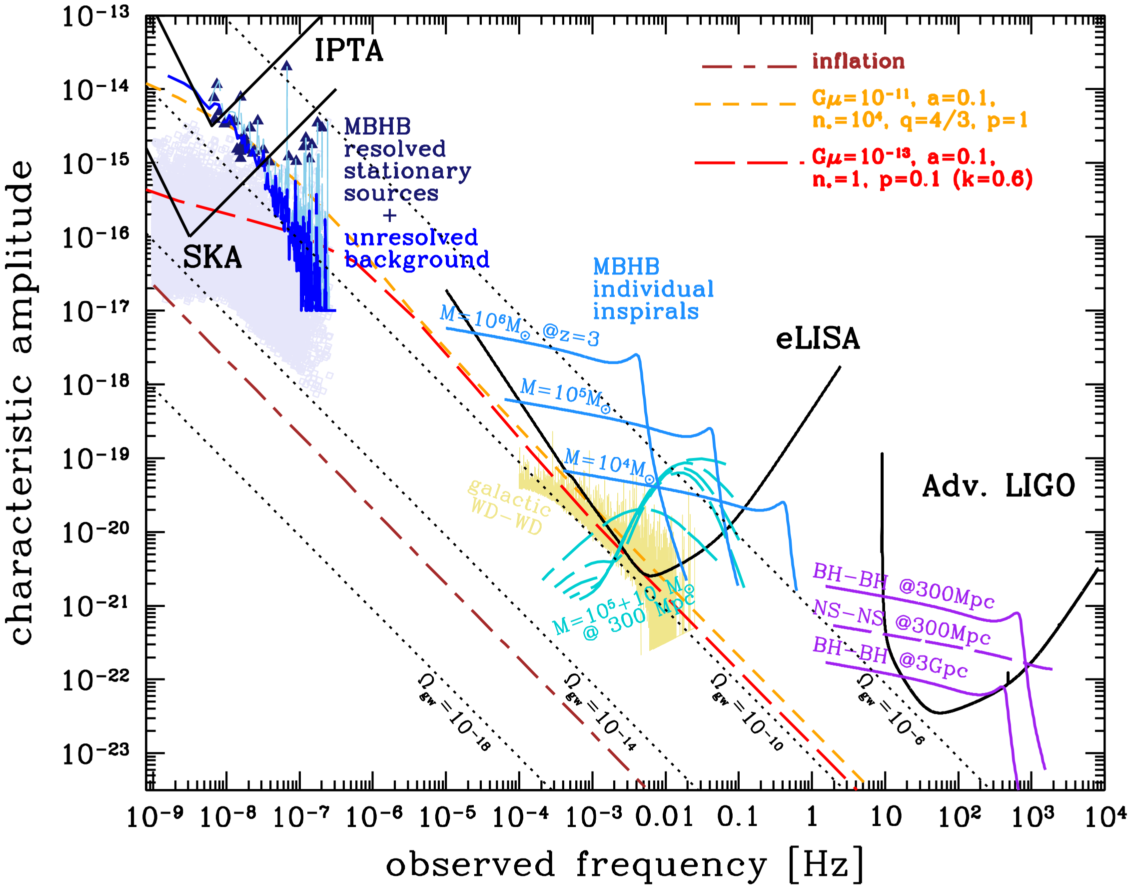

Figure 4 – Characteristic strain h_c of gravitational waves as a function of frequency. The expected sensitivities of the IPTA and of the SKA are plotted, along with realizations of the expected GW signal from the cosmological population of supermassive black hole binaries. The brown, red and orange lines correspond to expected cosmological backgrounds from standard inflation and selected string models, with the parameters displayed at the top right. Also shown are the eLISA sensitivity curve and the expected sensitivity curve of Advanced LIGO. See [26] for full details.

The continental and international PTAs have, until now, not been able to detect any low-frequency GW signal in the pulsar timing data. A number of upper limits on the amplitude of a gravitational wave background have been published (see e.g. [21, 22, 23]). One of the most stringent upper limits placed to date is that published by the IPTA, of 1.7×10^{-15} [24]. IPTA limits on a stochastic background of gravitational waves are plotted in Figure 4, along with the expected sensitivity curves for the SKA, eLISA and Advanced LIGO. Numerous other upper limits have been placed on GW emission from individual supermassive black hole binary systems, or GW bursts with memory. As can be seen from Figure 4, it is expected that future radio telescopes, such as the SKA, will increase our sensitivity of low-frequency GWs significantly. New generation instruments will indeed enable us to monitor known stable MSPs with much increased sensitivity and thus extract more precise and more numerous TOAs. Additionally, they will allow us to probe the Galactic population of MSPs much more deeply, thereby increasing the number of ultra-stable MSPs that PTAs could monitor. It is expected [25] that 5 to 10 years of timing of at least 20 MSPs with an accuracy better than 100 ns would be needed to make a first detection of the GW background. At present, this level of accuracy is achieved on a handful of MSPs only; hence the need to discover many new stable MSPs. However, current PTA experiments also need to address a number of limitations to increase their sensitivity to gravitational waves. In the following section we discuss some of the main known limitations of GW searches with PTAs.

3 Current limitations

As mentioned in the previous section, currently-existing PTAs have not yet been able to detect significant GW emission in the $\sim 1 - 100$ nHz frequency range. One obvious limitation in current searches for GWs by PTAs is that the number of MSPs that can be timed with very high accuracy (typically, with an RMS timing residual < 100 ns over a decade or more) is limited to a handful of objects at present. Future pulsar searches with new generation radio telescopes such as FAST, MeerKAT or SKA will likely expand the known population of ultra-stable MSPs. With their increased collective area and frequency bandwidths, future instruments will also measure pulsar TOAs much more accurately. Denoting σ the uncertainty on a given TOA, we have:

$$\sigma \propto \frac{w}{S_{\rm PSR}} \times \frac{T_{\rm sys}}{A} \times \frac{1}{\sqrt{B \times t_{\rm obs}}}. \tag{5}$$

In the above expression, w is the pulse width of the considered pulsar, $S_{\rm PSR}$ is its flux density, $T_{\rm sys}$ is the telescope's system temperature, A its collecting area, B is the frequency bandwidth and $t_{\rm obs}$ denotes the duration of the observation.

Interestingly, for some bright MSPs the sensitivity of current telescopes is not a limiting factor for timing precision. One key assumption of current pulsar timing techniques is that pulsar pulse profiles are stable when averaged over a large enough number of rotations. Under this assumption, pulsar profiles are averaged and compared to a standard profile for extracting TOAs, which then refer to a common phase in the pulse profile. For a small number of MSPs such as PSR J1713+0747, intrinsic fluctuations in the phase and amplitude of the pulse components are observed, a phenomenon known as "pulse jitter" (see e.g. [27]). Pulse jitter is a source of white noise that limits timing accuracy for the brightest MSPs only. However, with their much increased sensitivity compared to current instruments, future telescopes are expected to be very affected by jitter noise. A way to limit the effect of jitter is to increase the duration of observations, so that phase and amplitude fluctuations get averaged.

In addition to this source of noise acting on short timescales, MSPs are also affected by long-term timing perturbations referred to as "timing noise". These perturbations are currently not well understood and are most prominent in pulsars with long data spans and high-enough instantaneous timing accuracy. Various methods exist for whitening the white noise and the red noise introduced by those long-term intrinsic instabilities.

Other sources of noise affecting pulsar timing on both short and long timescales are those caused by changes in the interstellar plasma, between pulsars and the Earth. Radio waves are dispersed by the free electrons along the line-of-sight between the observed pulsar and the telescope. This dispersion effect induces a time delay given by:

$$\Delta t_{\rm DM} = D \times \frac{\rm DM}{f^2}, \tag{6}$$

where f is the observing frequency, $D = 4.148808 \times 10^3$ MHz^2 pc^{-1} cm^3 s is the dispersion constant and DM, the "dispersion measure", is the integrated column density of free electrons: $\mathrm{DM} = \int_0^d n_e \, dl$. The latter quantity is observed to vary as the Earth and the pulsars move with respect to the interstellar medium. As a consequence of this DM variation, the timing residuals of MSPs are affected by frequency-dependent delays that can be mitigated by observing pulsars at different radio frequencies, or by using wide-band frequency receivers. Pulsar observations at low radio frequencies (*i.e.*, $100 - 400$ MHz or below) by instruments such as LOFAR or NenuFAR are particularly useful in this regard, since they are strongly affected by dispersive effects and can thus be used to monitor and study turbulences in the interstellar medium. Additionally, multi-path propagation in the inhomogeneous interstellar plasma can cause radio pulses from pulsars to get broadened. The time-varying delays introduced by this pulse broadening are another source of noise that need to be mitigated when analyzing PTA data.

Unlike the above sources of noise, which are generally uncorrelated between pulsars, some other processes can introduce correlated noise in the timing data. Understanding and mitigating correlated noise is of prime importance, since, as mentioned above, PTAs search for low-frequency gravitational wave signals by analyzing correlations in the timing residuals of arrays of pulsars. Sources of correlated noise are, for example:

- Errors in terrestrial clocks, which are expected to affect TOA measurements of all pulsars identically.
- Inaccuracies in solar system ephemerides, which are used to convert topocentric TOAs to the solar system barycenter.
- Variations in the solar wind, which induce frequency-dependent timing residuals.

The Nançay Radio Telescope plays a key role in the search for GWs with PTAs, by recording large volumes of high-quality pulsar timing data which are shared and analyzed in the context of the EPTA and of the IPTA.

The current pulsar instrumentation in place at the Nançay Radio Telescope is the "NUPPI" backend, which has been used as the main pulsar instrument since mid-2011. Nançay observations of pulsars are mainly done at 1.4 GHz, with observations also done at higher frequencies of 2.1 or 2.5 GHz, to be sensitive to the short and long-term DM variations mentioned above. The NUPPI backend coherently dedisperses the radio signal over a total instantaneous frequency bandwidth of 512 MHz. The pulsar team in Orléans is currently involved in the construction of a new backend, that will enable pulsar observations over a total frequency bandwidth of ~ 2 GHz. Such large instantaneous frequency coverage will be particularly useful for mitigating several of the above-listed sources of noise.

The list of noise processes given above is non exhaustive. Current PTAs are actively working on understanding these sources of noise and defining analysis or observation strategies for mitigating them as much as possible. Despite these limitations, PTAs have already obtained stringent limits on GW emission from the population of supermassive black hole binaries, and have started to put some strong constraints on the properties of this population. With their much increased sensitivity compared to current telescopes, future large radio telescopes such as the SKA will make it possible to monitor pulsars with very high timing accuracy and will also discover many new stable MSPs. Although challenging, PTA-type GW searches with future instruments will undoubtedly open a new window on the study of GWs at low frequencies.

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