Efficient Full Wave Simulation of SKA-low Stations
Simulation ondes pleines efficace pour les stations SKA basses fréquences

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Abstract:
Fast and accurate methods to compute the mutual coupling between elements and with a finite ground plane are presented in this paper. The initial formulation is based on the Method of Moments (MoM). The HARP software presented here accelerates the MoM solution by using a combination of the Macro Basis Function (MBF) approach with an interpolatory technique. After that, those MBFs, which correspond to an infinite ground plane solution, are used to describe the interactions of the array with a finite ground plane lying on a semi-infinite soil. The methods are validated here for the SKA Log Periodic antenna (SKALA), under study for the Square Kilometer Array (SKA). The patterns are compared here with results obtained with the commercial software FEKO.

Résumé:
Cet article présente deux méthodes permettant de tenir compte du couplage mutuel inter-éléments ainsi qu'avec le plan de masse fini dans les réseaux d'antennes. La formulation initiale est basée sur la Méthode des Moments. Celle-ci est accélérée grâce à une combinaison de la méthode des Macro fonctions de base (MBF) avec une technique interpolatoire dans le software baptisé HARP. Ces MBFs, calculées sous la condition d'un plan de masse infini, sont utilisées pour caractériser les interactions avec un plan de masse fini lui-même posé sur un sol infini. Les méthodes sont ensuite appliquées à l'antenne SKALA (SKA Log Periodic antenna) du radiotélescope Square Kilometer Array (SKA). Les diagrammes de rayonnement sont ensuite validés à l'aide des résultats obtenus avec le software commercial FEKO.

1 Introduction

The SKA (Square Kilometer Array) [1] represents the next generation of radio telescopes. It aims to survey the sky much faster and in a more accurate way than any other previous system. Therefore, the SKA will collect information over more than a square kilometer area. This project, in which more than 100 companies and research institutions from different countries cooperate, represents a true challenge for engineers in many areas such as digital hardware, signal processing, antenna design and array simulation. This radio telescope aims to achieve an unprecedented sensitivity by using thousands of dishes and up to a million of low-frequency antennas. The latter will form many base stations appearing as a collection of radio telescopes. The main targeted cosmology experiments are the Epoch of Reionization (EoR) and the Cosmic Dawn (CD) [2]. The SKA low-frequency array is composed of base stations containing each 256 SKA Log-periodic Antennas (SKALA [3]). The frequency band of this array lies between 50 MHz and 350MHz. A base station using the fourth version of the SKALA antenna has already been deployed in Australia and is currently under test. The deserts of South Africa and Australia have been designated for the SKA location in order to benefit from a minimum radio interference.

In order to perform the best calibration, radio-astronomers need accurate Embedded Element Patterns (EEP), i.e. the radiation pattern of each individual element assuming the other antennas are passively terminated. Indeed, since SKA-low is a phased array and given that the embedded element patterns are not the same, the shape of the beam changes rapidly as the array is scanned. Small changes in the shape of the beam are not necessarily an issue, since some imaging algorithms can cater for different patterns at the level of different stations. Though, since high dynamic range is targeted, those small differences in array patterns need to be known very accurately. Regarding the sidelobes, things are even more crucial because non-regular arrays produce relatively high sidelobes and the impact of mutual coupling (i.e. of varying embedded element patterns) is actually much higher in the sidelobes. Hence, if one wants to perform nulling of far-out interferrers [4], the accurate knowledge of embedded element patterns is even more crucial.

The mutual coupling and the ground plane finiteness strongly impact the EEPs. In order to accurately predict these patterns, full wave simulations such as those obtained with the Method of Moments [5] need to be carried out. When considering large irregular arrays, full wave simulations using commercial software is time consuming and requires a lot of memory. This issue has been tackled for decades and two categories of methods appear. The first category represents the iterative methods based on Multipoles [6]. In those methods, a new simulation is needed every time the excitation port changes and for every change of the antennas positions. Moreover, condition number issues can appear due to fine-mesh details. The other category contains the non-iterative solvers. These methods are based on the assumption that the current distribution on the antennas can be decomposed into a limited number of current distributions. In this category, the Macro Basis Function [7] technique allows to solve smaller systems of equations. Besides, fast methods have been developed to compute the interactions between MBFs [8], [9], [10]. However, when considering a finite ground plane, the interaction of the MBFs with the latter become prohibitive. The inhomogeneous plane wave algorithm can be used to efficiently perform the interactions between scatterers [12]. In this spectral approach, the number of plane waves required to compute the interactions greatly decreases with increasing distance between the scatterers.

In this paper, a software named HARP [13] which efficiently handles the mutual coupling is presented. It is based on a combination of the MBF method with an interpolatory technique [10]. The set of MBFs is built assuming an infinite ground plane. The finite ground plane is then taken into account by computing the interaction with the MBFs using the inhomogeneous plane waves algorithm.

The remainder of the paper is organized as follows: Section 2 briefly describes the main design parameters of the SKALA antenna. Section 3 describes the HARP software and compares it in terms of performance to the commercial software CST [11]. Section 4 describes how the finite ground plane is taken into account using inhomogeneous plane waves and Section 5 concludes the paper.

2   SKALA

The SKALA [3] shown in Figure 1 has been designed according to the observation of those predetermined cosmology experiments: the Epoch of Reionization (EoR) and the Cosmic Dawn (CD). Based on these experiments, the SKALA should present a smooth frequency response in order to detect faint signals. The antenna has been designed to maximize the sensitivity of the SKA-low array on a 7:1 frequency band by optimizing the effective area, minimizing the footprint and the receiver noise. In order to measure the foregrounds of the EoR, the antenna also requires a low relative cross-polarization. An evaluation of the SKALA performance with respect to the EoR and CD experiments is presented in [14]. According to the planned number of deployed antennas (more than 3 million), the cost per element and the durability are also important parameters. Indeed, the antenna should be able to last more then 30 years in the desert.
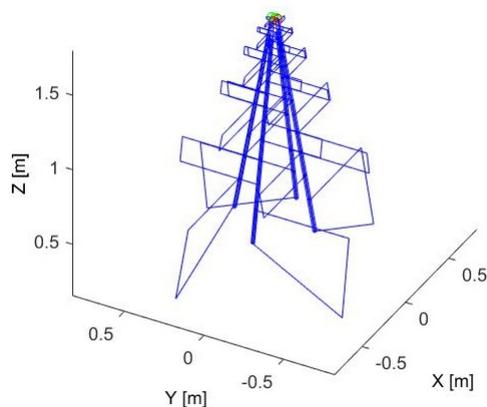

Figure 1 − Verison 2 of SKALA

3   HARP

The traditional Method of Moments (MoM) gives the current distribution on the antennas as the solution of the following system of equations:

$$Zi = v \tag{1}$$

2

where Z is the MoM impedance matrix, v is a vector of excitation and i is the current distribution. Considering N_a antennas meshed with N_e basis functions, the solution has a complexity $O((N_a N_e)^3)$.

In the MBF formulation, one considers a Q matrix where each column corresponds to a MBF. Thanks to that consideration, the solution, approximated as $i^r \simeq Q\, i$, can be obtained by solving a reduced system of equations [15]:

$$Z^r i^r = v^r \tag{2}$$

where $Z^r = Q^H Z Q$ and $v^r = Q^H v$. Thanks to that formulation, the solution complexity is reduced to $O((N_a N_m)^3)$ where N_m is the number of MBFs which is considerably smaller than the number of basis functions N_e. As an example, the SKA-low stations can be simulated with $N_m = 20$ MBFs whereas every antenna needs to be meshed with $N_e = 1218$ basis functions.

The blocks of the matrix Z^r contain the interactions between MBFs. The far-field interaction between a MBF S placed at the origin and a MBF T positionned at $(r_{mn}, \hat{\alpha})$ can be approximated as:

$$Z_{TS}^{app}(r_{mn}, \hat{\alpha}) \simeq -j\omega\mu\ \overrightarrow{F}_{T,m}^{\hat{\alpha},*} \cdot \overrightarrow{F}_{S,n}^{\hat{\alpha}}\ \frac{e^{-jkr_{mn}}}{4\pi r_{mn}} \tag{3}$$

where $\overrightarrow{F}_{T,m}^{\hat{\alpha},*}$, and $\overrightarrow{F}_{S,n}^{\hat{\alpha}}$ are the radiation patterns of the testing and source MBFs respectively, $*$ stands for the complex conjugate, j is the wavenumber, ω is the angular frequency and μ is the free-space permeability.

The interpolatory method used in HARP was proposed in [10]. In this method, the blocks of the matrix Z^r are obtained as a combination of the far-field interaction in (3) and a matrix B represented by a harmonic-polynomial model.

This matrix B can be obtained in three steps:

- Subtraction of the far-field expression for the interactions.

- Phase extraction

- Harmonic polynomial representation

Applying the two first steps gives:

$$B_{TS}(r_{mn}, \hat{\alpha}) = \frac{Z_{TS}^r(r_{mn}, \hat{\alpha}) - Z_{TS}^{app}(r_{mn}, \hat{\alpha})}{e^{-jkr_{mn}}} \tag{4}$$

The subtraction in the numerator represents the far-field subtraction and the division indicates the phase correction step. Note that the method requires to compute the exact interactions on a limited number of points positioned on a pre-defined polar-radial grid.

After applying the change of variable $d = \frac{1}{r^2}$, (5) can be fitted with the following harmonic polynomial model:

$$B_{TS}(r_{mn}, \hat{\alpha}) = \sum_{p=-P}^{P} e^{jp\alpha} \sum_{q=0}^{Q} c_{pq}\ d^q \tag{5}$$

where, c_{pq} are the coefficients calculated in the least-squares sense, P is the Fourier Series order and Q is the polynomial order.

Thanks to this interpolatory method, the interaction between MBFs can be fast computed as:

$$Z_{TS}^r(r_{mn}, \hat{\alpha}) = Z_{TS}^{app}(r_{mn}, \hat{\alpha}) + B_{TS}(r_{mn}, \hat{\alpha})\ e^{-jkr_{mn}} \tag{6}$$

Let us consider an array of 16 SKALA antennas on an infinite ground plane. The radiation patterns obtained with FEKO [20] and HARP are compared in Figure 3 considering a frequency of 145 MHz.

Some differences may appear in the sidelobes due to the different modelling of the antenna in HARP and FEKO. Moreover, the simulations are done using the fourth version of the SKALA antenna which is more complicated than the previous versions. Considering now a SKA station composed of 256 version 2 SKALAs, the commercial software needs more than 90 hours to simulate the station [13]. Moreover, a new simulation is required every time the excitation port changes. The software HARP, however needs 3 hours of pre-processing per frequency. Once the pre-processing is done, HARP only needs 0.5 min to simulate the station. If the antennas positions are modified, the simulation only takes another 0.5 min.
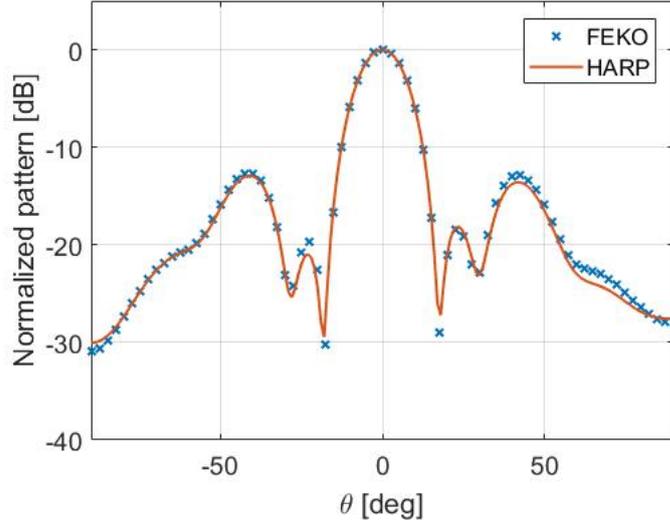
Figure 2 − E-plane normalized radiation patterns of HARP vs FEKO

4   Finite ground plane

As stated before, not only the mutual coupling impacts the embedded element patterns but also the finite ground plane as shown in [16]. In this section, the inhomogeneous plane waves algorithm is used to describe the interactions between the MBFs calculated thanks to the HARP software and the finite ground plane. Using a MoM formulation, the finite ground plane is meshed with RWG basis functions [19]. It should be pointed out that the MBFs are calculated assuming an infinite ground plane. However, as will be shown in the results, this assumption is consistent.

Let us consider a SKALA above a finite ground lying itself upon a semi-infinite soil. Using the mathematical definitions and derivations in [17]. The field radiated by the antenna and tested by the finite ground plane can be written as:

$$\overrightarrow{E}(x,y,z) = \frac{-jk\eta}{(2\pi)^2} \iint F_p(k_x,k_y) \cdot \hat{e}_p \, (1+\Gamma_p) \, \frac{e^{-j(k_x x + k_y y - k_z z)}}{2jk_z} \, Q \, dk_{xR} \, dk_{yR} \tag{7}$$

where p is the polarization (TE or TM), η is the free space impedance, k is the wavenumber which is also the norm of the wavevector $\overrightarrow{k} = (k_x, k_y, k_z)$, Γ_p is the reflection coefficient due to the presence of the soil [18], Q is a term allowing the integration only on real parts of k_x and k_y and $F_p(k_x,k_y)$ is the radiation pattern of the MBF expressed as:

$$F_p(k_x,k_y) = \sum_{m=0}^{M} \epsilon_m \iiint_V \overrightarrow{J_m}(\overrightarrow{r}') \cdot \hat{e}_p \, e^{j(k_x x' + k_y y' - k_z z')} \, dV \tag{8}$$

where M is the number of elementary basis functions per MBF, ϵ_m is the weight of the mth basis functions and $\overrightarrow{J_m}(\overrightarrow{r}')$ is the mth basis function of the MBF situated in $r' = (x', y', z')$. Once the tested field in (7) has been computed, the current distribution on the ground plane is obtained by solving the following system of equations:

$$Z_{gg} \, i_g = v \tag{9}$$

where Z_{gg} is the MoM impedance matrix of the ground, i_g represents the equivalent currents of the ground plane and v is the excitation vector resulting from the projection of (7) on the finite ground basis functions.

Let us consider ρ the radial distance from the antenna. It should be pointed that computing the interactions with the finite ground plane is very efficient when ρ is lower than the wavelength. However, the required number of inhomogeneous plane waves becomes too high when ρ is increasing above a few wavelengths and the methods looses its efficiency. In a nutshell, it appears that the inhomogeneous plane wave algorithm is efficient only when considering a relatively low field of view defined as ρ/z' where z' is the distance between the antenna MBF and the ground plane [21].

As a validation, let us consider a SKALA-2 antenna lying above a 8 m diameter finite ground plane. The frequency considered is 80 MHz. The results are consistent with those of FEKO and confirm the antenna infinite ground plane currents assumption.
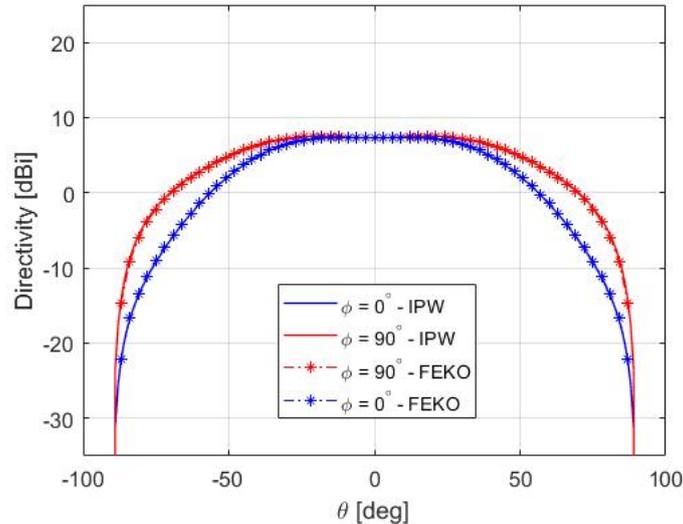
Figure 3 − Radiation patterns of HARP vs FEKO considering one SKALA on a 8 m diameter ground plane

5   Conclusion

In this paper, we have described efficient methods to compute the coupling between elements as well as with the finite ground plane. The results have been then validated thanks to the commercial software FEKO. However, the inhomogeneous plane waves methods do not perform efficiently when the finite ground plane becomes electrically large. Thus, the door is open to new methods handling large ground planes.

6   References

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